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LEVEL II

PROBABILITY DISTRIBUTION

OF IMPACT FOR A SATELLITE

BOOSTER AFTER LOSS OF CONTROL

THESIS

AFIT/GA/AA/78D-1 Scott W. Benson
2nd Lt, USAF

(14) AFIT/GA/AA/78D-1

PROBABILITY DISTRIBUTION OF IMPACT FOR A SATELLITE BOOSTER AFTER LOSS OF CONTROL.

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air Training Command
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Scott W. Benson, B.S.
2nd Lt USAF
Graduate Astronautical Engineering

March 1979

Approved for public release; distribution unlimited.

Acknowledgement

I wish to thank my wife, Pam, for her understanding and her persistence in keeping me at work. I also wish to thank my advisor, Capt. Bill Wiesel, for his advice and patience towards the completion of this study.

Scott W. Benson

Contents

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List of Symbols

Symbol	Definition
a	Semi-major axis of conic section
e	Eccentricity of conic section
E	Probability density
E_i	Eccentric anomaly at impact
E_o	Eccentric anomaly at failure
g	Gravitational acceleration
h	Altitude
$\vec{h}$	Angular momentum
m	Mass
p	Parameter of conic section
P	Probability
$\vec{r}$	Position of vehicle
R_{PER}	Perigee height of conic section
R_E	Radius of earth
t	Time
t_B	Rocket burn time
T	Thrust
$\vec{V}$	Velocity of vehicle
V_{CIRC}	Velocity to obtain circular orbit
V_E	Equivalent exit velocity of rocket
V_R	Residual velocity
x	Range
α	Azimuth
β	Mass flow rate
γ	Flight path inclination

Symbol	Definition
δ	Elevation
θ	Longitude
ι	Angle of inclination
μ	Earth gravitational constant
ν_i	True anomaly at impact
ν_o	True anomaly at failure
ϕ	Latitude
ψ	Angle traversed by vehicle

Abstract

The impact area and regions of high impact probability after booster failure were found for a sample space system. The impact footprints were found to be drop-shaped. They increased in size as the residual velocity vector elevation approached the original flight path, or when the booster failure was assumed earlier in the boost trajectory. Probability densities at impact were found, and plotted against the longitude of impact. Singularities in the probability density were discovered. The singularities correspond to areas of high impact probability, and occur when a maximum latitude or longitude is obtained at a fixed residual velocity vector azimuth or elevation. These singularities were plotted along the impact envelopes. A sample failing booster system was used to demonstrate the impact area and probability characteristics.

PROBABILITY DISTRIBUTION OF IMPACT FOR
A SATELLITE BOOSTER AFTER LOSS OF CONTROL

I. Introduction

Background

In light of the recent Soviet satellite reentry and impact in Canada, it is becoming more important to determine the impact areas and probabilities of failing space systems. The main reasons for concern are the possibilities that there are nuclear materials on board, in which personal safety is endangered or that the system is highly classified, in which national security is endangered. Of special interest to the United States are the slightly retrograde orbits launched from Vandenburg AFB, California. These type of orbits pass over Moscow in the first revolution, and might be cause for concern in the case of booster failure.

The objective of this study was to demonstrate a method of predicting impact areas and probabilities for a failing booster system. A booster is launched and, at some time before orbit injection, suffers total loss of control. The vehicle follows a new trajectory to earth impact. The position of impact and the probability is calculated for each possible failure. The launch to impact trajectory is represented in Fig 1.

Approach

The booster equations of motion were first integrated to a circular orbit. This was accomplished by setting final

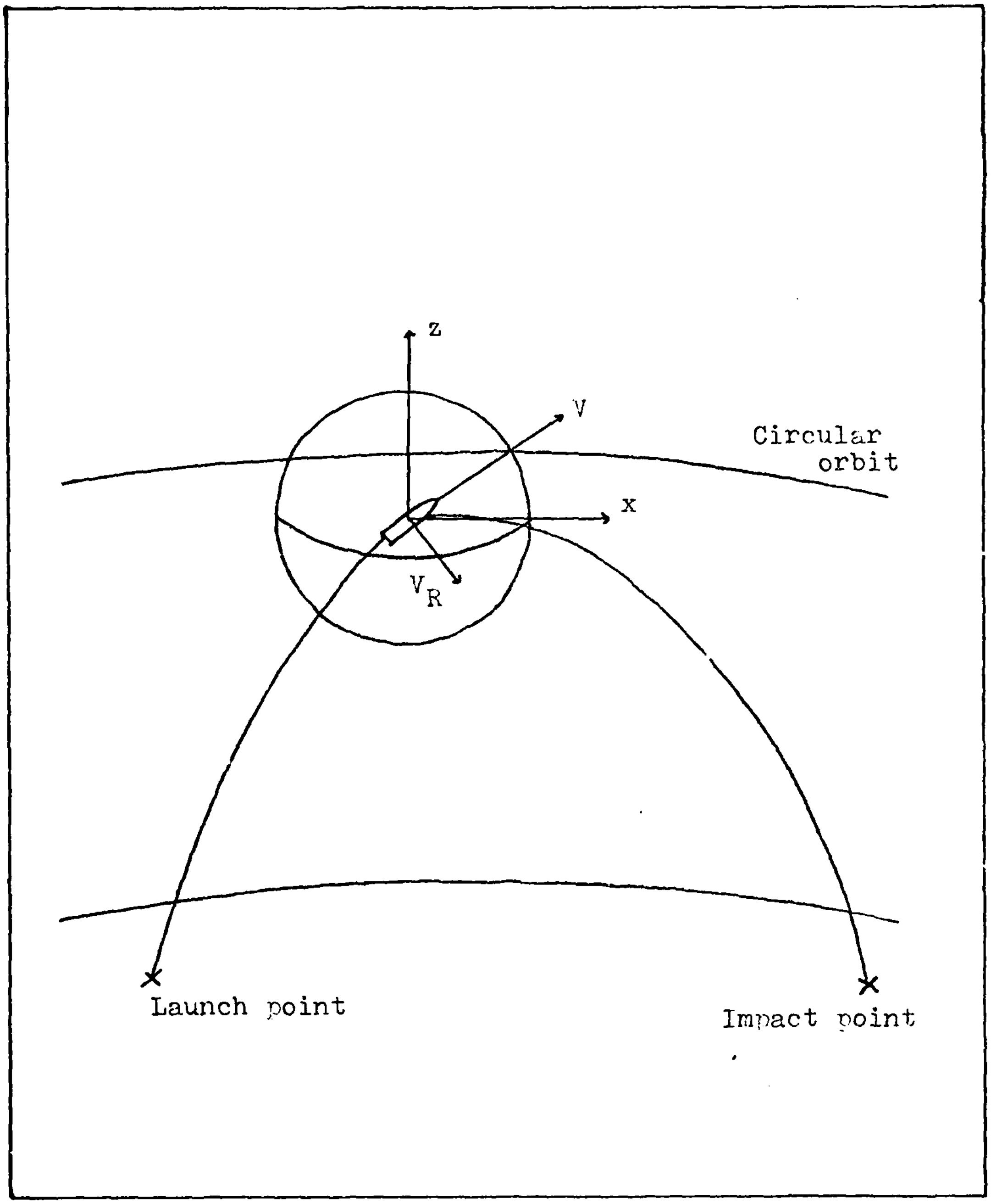

Fig. 1. Launch-to-Impact Trajectory

orbit conditions and solving the boundary value problem. Once a launch-to-orbit trajectory was obtained, the equations of motion were integrated to several times lower than the time to orbit injection. At this point, the residual velocity was applied in one of many failure directions. The vehicle position and new vehicle velocity were obtained and the orbit they define was found. From the orbital elements, it was determined when and where the vehicle impacted the earth. Knowing the impact point, the probability density at impact was found by propagating the probability density at failure, through the new trajectory, to the ground.

Scope

For this initial study, the scope of the problem was limited. The booster was assumed to launch easterly along the equator, through a vacuum, to an arbitrary circular orbit height. The booster was modeled as a simple single stage booster, with a constant thrust profile. The number of possible failures was limited to a small, representative sample. There are several other less important assumptions made throughout the body of this report.

The following chapter addresses the problem solving procedure in detail.

II. Procedure

Launch to Orbit Injection

The beginning stategy was to find a launch trajectory that terminated in a circular orbit. Obtaining this trajectory requires the solution of a boundary value problem, using the equations of motion for a gravity turn trajectory (Ref 5:335-43).

$$\dot{x} - V\cos\gamma = 0$$

$$\dot{h} - V\sin\gamma = 0$$

$$\dot{V} + g\sin\gamma - \frac{\beta V_E}{m} = 0$$

$$\dot{\gamma} + \frac{g\cos\gamma}{V} = 0$$

$$\dot{m} + \beta = 0$$

$$T = \beta V_E$$

In these equations, x and h are the vehicle range and altitude respectively, V is the velocity magnitude, γ is the flight path inclination with respect to the local horizontal, m is the vehicle mass, β is the rocket mass flow rate, and V_E is the rocket equivalent exit velocity. The launch phase is illustrated in Fig 2.

There are several underlying assumptions used in obtaining this set of equations. First, the trajectory is relative to a normal reference frame; therefore, the local earth must be assumed to be flat. Second, flight is assumed to be in a vacuum, negating the lift and drag effects. Thirdly, the thrust is assumed to act parallel to the velocity.

Other assumptions were made to simplify the boundary

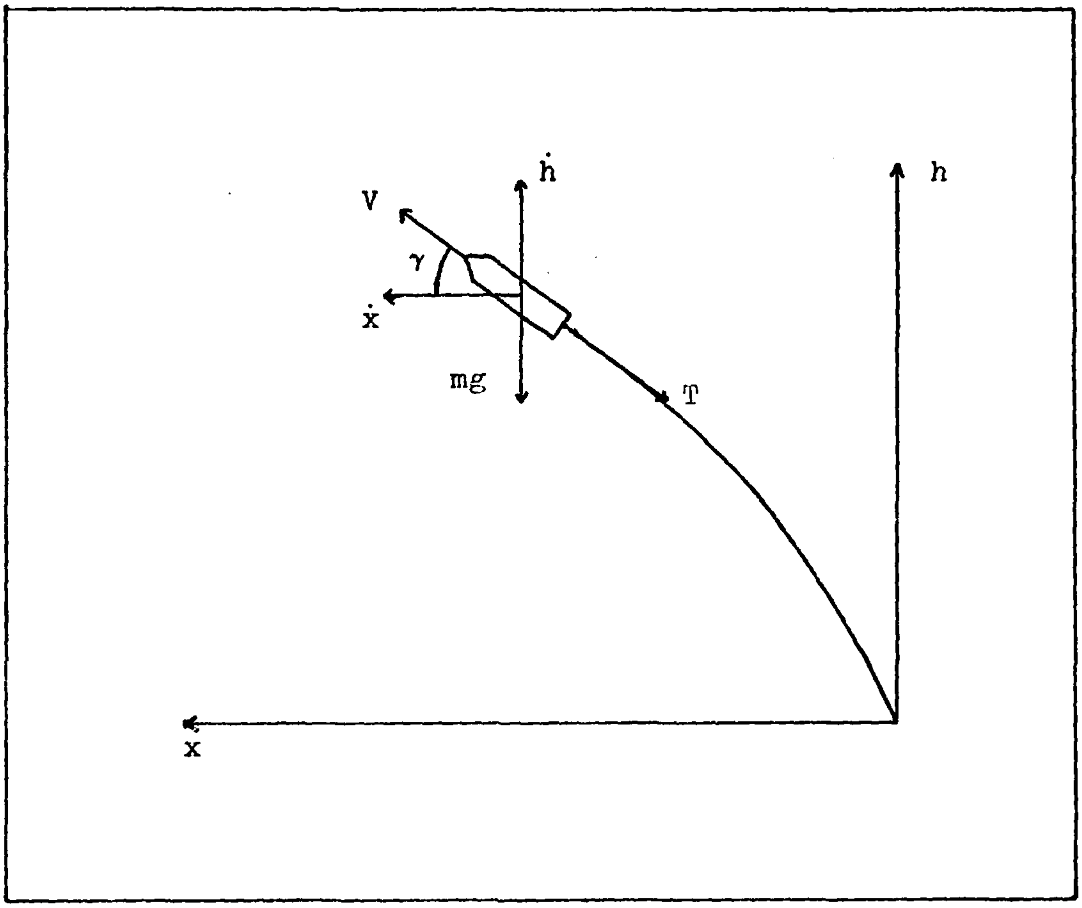

Fig. 2. Booster Launch

value launch problem. The mass flow rate, β, and the equivalent exit velocity, V_E, were assumed to be constant, resulting in a constant thrust profile. The gravitational acceleration, g, was also assumed to be constant.

The boundary value launch was executed as follows. An initial mass, and appropriate values of thrust, mass flow rate, and equivalent exit velocity were chosen. The equations of motion were integrated through a number of different burn times, and the resulting flight path inclinations

were obtained. The integration routine used was ODE, an ordinary differential equation integrator found in the IMSL subroutine package. The burn time to zero path inclination, when the booster is parallel to the earth, was found using the Aitken method of interpolation. The Aitken subroutine, INTERP, is also found in the IMSL library. Integrating again to this burn time gives values for path inclination, velocity and altitude. To achieve successful circular orbit, the final velocity must be equal to the circular orbit velocity at that altitude. The initial value of mass was changed to obtain this required boundary condition.

The method used in changing the initial condition, mass, is discussed here. Rearranging the equations of motion gives an equation for instantaneous change in velocity for a change in mass.

$$\frac{dV}{dm} = \frac{-T}{\beta m} + g\frac{\sin\gamma}{\beta}$$

The required change in velocity is

$$\Delta V = V - V_{CIRC}$$

where

$$V_{CIRC} = \sqrt{\frac{\mu}{R_E + h}}$$

The first change in mass becomes

$$\Delta m = \frac{-\Delta V}{\frac{dV}{dm}}$$

This change in mass was added a number of times; the equations of motion being integrated to a horizontal flight path for each mass increment. Again, the Aitken interpola-

tion scheme was used to find the initial value of the total mass which satisfied the boundary conditions.

The interpolated values of mass and burn time were then used to integrate the equations of motion to the final set of parameters.

Booster Failure

Booster failure was modeled by an instantaneous change in direction of the residual velocity at a given time in the booster trajectory.

A time decrement was chosen and subtracted from the circular orbit trajectory burn time. The equations of motion were integrated to the resultant time, which gave a set of parameters before orbit injection. The magnitude of the residual velocity was found by subtracting the final velocity from the circular orbit velocity.

$$V_R = V_{CIRC} - V$$

V_{circ} at initial alt. – failed or original ?

It was assumed that the direction of the residual velocity could be towards any point on a sphere surrounding the vehicle. For this study, the direction was incremented by 30° in azimuth and elevation, to give a total of 62 possible directions. The increment was decreased later in the study to achieve a better understanding of the probabilities of impact. The sphere of points was defined with the polar axis along the local vertical, as seen in Fig 3.

The components of the residual velocity can easily be found, as shown in Fig 4.

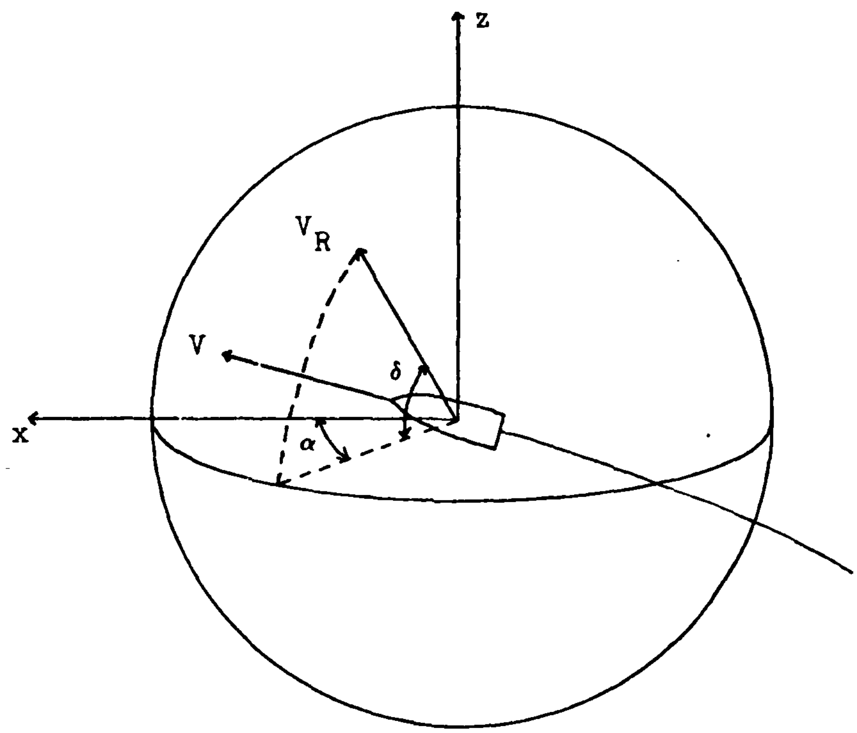

Fig. 3. Booster Failure

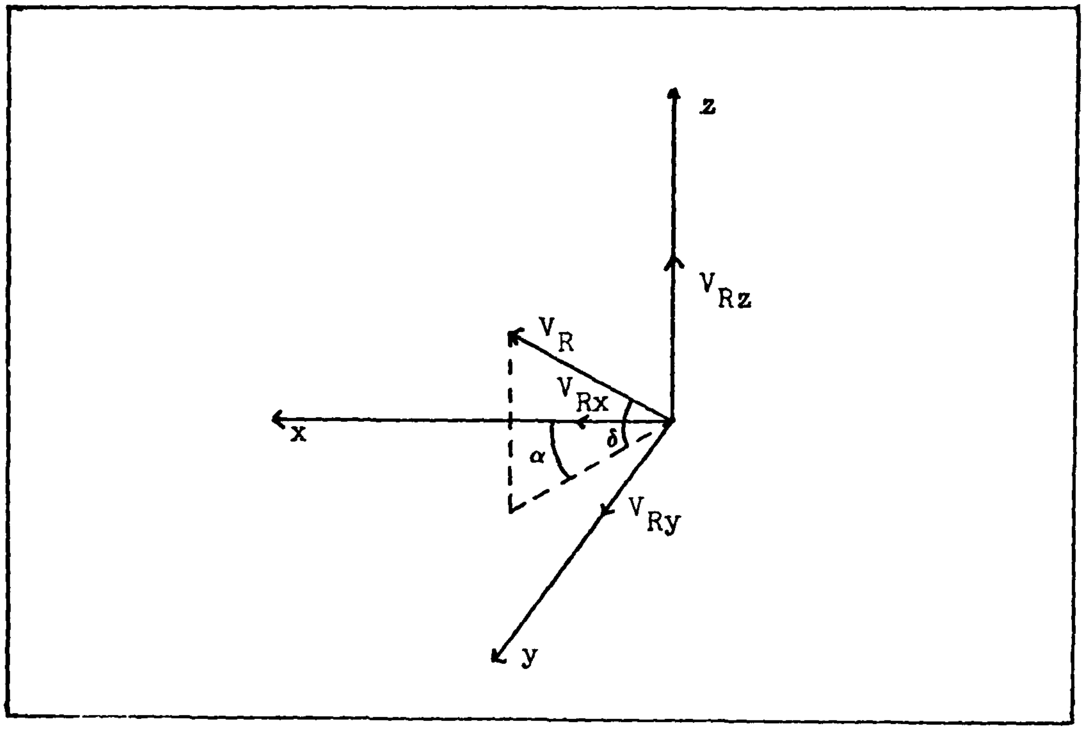

Fig. 4. Residual Velocity Components

$$V_{Rx} = V_R \cos\alpha \cos\delta$$

$$V_{Ry} = V_R \sin\alpha \cos\delta$$

$$V_{Rz} = V_R \sin\delta$$

where

the z axis is aligned along the local vertical
the x axis is in the plane of the original flight path

These components were then added to the flight velocity components, $\dot{x}$ and $\dot{h}$, at burnout to give a resultant velocity after booster failure, herein referred to simply as V. This velocity and the vehicle position, as found through the equation integration, gives the information needed to propagate the trajectory to impact.

Flight Trajectory Propagation to Impact

The requirements for this part of the study were to see if the booster impacts the earth, and if so, where it impacts and how long a time interval it covers before impact.

The method used in finding these answers starts with the conversion of the vehicle velocity and position after failure to the orbital elements. Using the orbital elements, it is simple to check if the vehicle impacts by checking the orbit perigee height. If it does impact, the angle traversed and the angle of deviation from the original flight path can be found. Knowing the angle traversed, the time of flight can be calculated, and the impact latitude and longitude are easily obtained.

The vehicle velocity and position after failure were converted to the necessary orbital elements(Ref 3:17-26).

$$\vec{h} = \vec{r} \times \vec{V}$$

$$\vec{e} = \frac{1}{\mu}\left[\left(V^2 - \frac{\mu}{r}\right)\vec{r} - \left(\vec{r}\cdot\vec{V}\right)\vec{V}\right]$$

$$p = \frac{h^2}{\mu}$$

$$a = \frac{p}{1 - e^2}$$

Using the calculated parameter and eccentricity, the orbit perigee height was found and checked against the earth radius, which is assumed to be constant.

$$R_{PER} = \frac{p}{1 + e}$$

check $$R_{PER} \le R_E$$

If the perigee height was greater than the earth radius, the

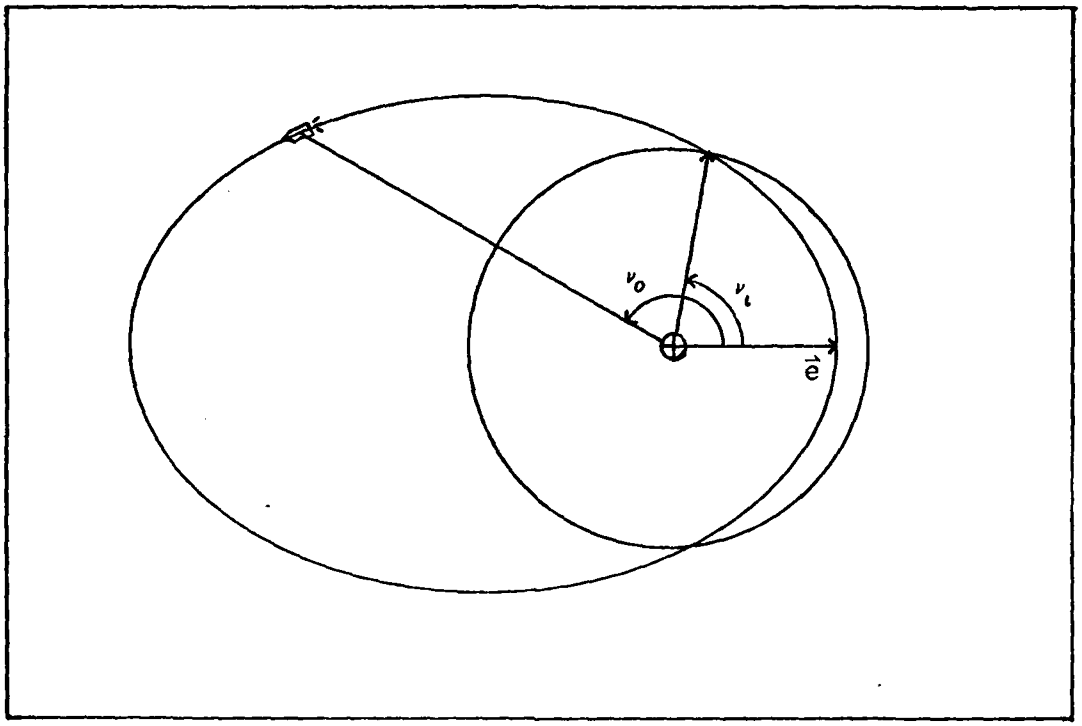

Fig. 5. Vehicle Orbit to Impact

vehicle would enter earth orbit, and those cases were ignored. If the perigee check indicated that impact did occur, the angle the vehicle traverses before impact, ψ, can be found by

$$\psi = 2\pi - \nu_0 - \nu_\iota$$

where

ν_0 is the true anomaly at failure
ν_ι is the true anomaly at impact, as seen in Fig 5

These true anomalies can be calculated using the equation of a conic section.

$$\nu_0 = \cos^{-1}\left(\frac{\frac{p}{r} - 1}{e}\right)$$

$$\nu_\iota = \cos^{-1}\left(\frac{\frac{p}{R_E} - 1}{e}\right)$$

The angle of inclination, ι, after failure is found by

$$\iota = \cos^{-1} \frac{\vec{h} \cdot \hat{k}}{\vec{h}}$$

where $\hat{k}$ is the unit vector along the z axis.

To obtain the time of flight from failure to impact, the eccentric anomaly, E, at each point of interest must be found.

$$E_o = \cos^{-1}\left(\frac{e + \cos \nu_o}{1 + e\cos \nu_o}\right)$$

$$E_\iota = \cos^{-1}\left(\frac{e + \cos(2\pi - \nu_\iota)}{1 + e\cos(2\pi - \nu_\iota)}\right)$$

The time of flight is found by solving Kepler's equation for each eccentric anomaly, and getting the difference in time.

$$\Delta t = \sqrt{\frac{a^3}{\mu}}\left[(E_\iota - e\sin E_\iota) - (E_o - e\sin E_o)\right]$$

This change in time represents the time from failure to impact. This time is added to the burn time to failure to obtain the total time of flight.

Using spherical trigonometry, the latitude, ϕ, and longitude, θ, are easily found from ψ and ι, as shown in Fig 6. The original simplifying assumption of launching along the equator takes effect here.

$$\phi = \sin^{-1}(\sin\iota \sin\psi)$$

$$\theta = \sin^{-1} \frac{\tan \phi}{\tan \iota}$$

In the cases when the change in velocity occurs in the plane of the original orbit, the latitude and longitude are simply

$$\phi = 0$$

$$\theta = \psi$$

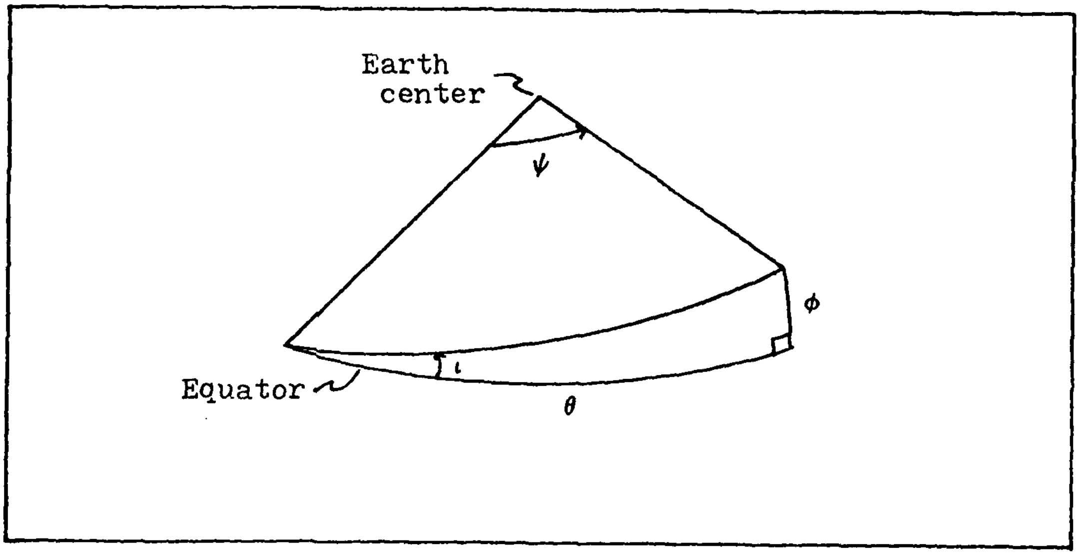

Fig. 6. Latitude and Longitude Traversed

The longitude can be added to the longitude covered prior to failure, θ_f, to obtain a total longitude. The failure longitude is found with the original flat earth assumption.

$$\theta_f = \sin^{-1} \frac{x}{R_E}$$

$$\theta_t = \theta + \theta_f \ .$$

At this point, a position and time of impact has been found for a vehicle which had been subjected to a velocity change before injection. The probability of impact at this position will now be determined.

Probability Determination

To determine the probability of impact at a specific latitude and longitude, the probability density at failure for the corresponding velocity change is found, then propagated along the trajectory to the ground, as seen in Fig 7.

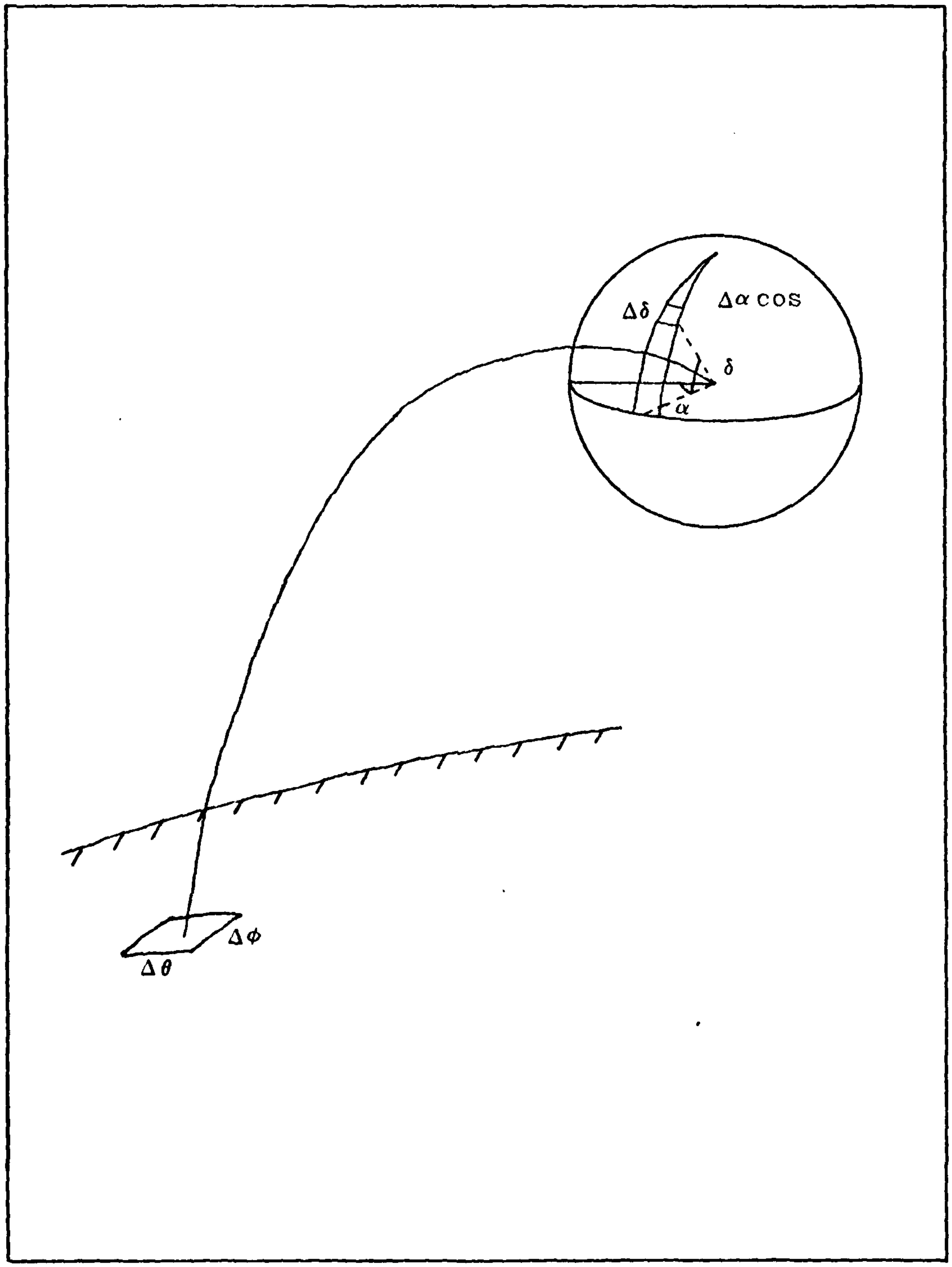

Fig. 7. Probability Propagation to Impact

For a small change in azimuth and elevation, the probability density at failure is

$$E(\alpha,\delta) = \frac{\cos\delta}{4\pi}$$

Therefore, the probability of the vehicle passing through an area of the sphere is

$$P(\alpha,\delta) = \int_{\alpha}^{\alpha+\Delta\alpha} d\alpha \int_{\delta}^{\delta+\Delta\delta} d\delta \; \frac{\cos\delta}{4\pi}$$

When integrated over the whole sphere (from $0 \rightarrow 2\pi$ in azimuth and $-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$ in elevation), the probability becomes unity.

Propagation of the probability to a latitude and longitude at impact results in

$$P(\phi,\theta) = \int_{\phi}^{\phi+\Delta\phi} d\phi \int_{\theta}^{\theta+\Delta\theta} d\theta \; \frac{\cos\delta}{4\pi} \; \frac{\partial(\alpha,\delta)}{\partial(\phi,\theta)}$$

The Jacobian is defined as a determinant.

$$\frac{\partial(\alpha,\delta)}{\partial(\phi,\theta)} = \begin{vmatrix} \frac{\partial\alpha}{\partial\phi} & \frac{\partial\delta}{\partial\phi} \\ \frac{\partial\alpha}{\partial\theta} & \frac{\partial\delta}{\partial\theta} \end{vmatrix}$$

As the changes in latitude and longitude become infinitesmal, the probability density becomes the integrand.

$$E(\phi,\theta) = \frac{\cos\delta}{4\pi} \begin{vmatrix} \frac{\partial\alpha}{\partial\phi} & \frac{\partial\delta}{\partial\phi} \\ \frac{\partial\alpha}{\partial\theta} & \frac{\partial\delta}{\partial\theta} \end{vmatrix}$$

In this study, the partial derivatives are estimated by numerical methods. Very small changes in azimuth and elevation are chosen and added. When the computer program is run, there are resulting small changes in latitude and longitude. The ratio of these changes becomes the numerical partial derivative.

$$E(\phi, \theta) = \frac{\cos \delta}{4\pi} \begin{vmatrix} \frac{\Delta \alpha}{\Delta \phi} & \frac{\Delta \delta}{\Delta \phi} \\ \frac{\Delta \alpha}{\Delta \theta} & \frac{\Delta \delta}{\Delta \theta} \end{vmatrix}$$

This probability density represents probability in relative magnitude. When the density is high, or approaching a singularity, the probability is high for that impact point.

The following chapter will discuss the resulting impact envelopes and probability density behavior for this study.

III. Results and Discussion

Gravity-turn Trajectory

To initiate the gravity-turn launch, the following parameters were chosen.

$$m = 21{,}000\ lb_m/g$$

$$\beta = .466\ lb_m/sec$$

$$T = 58{,}800\ lb_f$$

The launch-to-orbit trajectory had these final values.

$$t_B = 276.54996\ sec$$

$$x = 3078159.92872\ ft$$

$$h = 526417.72579\ ft$$

$$V = 25616.04328\ ft/sec$$

$$\gamma = .00000036\ rad \simeq 0$$

$$m = 398.14796\ lb_m$$

Impact Areas

Impact points for each possible failure in each time step were found. The plot in Fig 8 is a representative plot using one time decrement and elevations from $-\frac{\pi}{2} \rightarrow \frac{\pi}{2}$ radians in $\frac{\pi}{6}$ radian increments. Each curve of points represents a constant elevation of the residual velocity vector, with azimuth rotating through $\frac{\pi}{2}$ radians. It is seen that minimum latitude and longitude dispersions occur for elevations approaching the poles of the sphere of failure. The maximum dispersions occur for elevations equal to zero.

From the proximity of the impact points for each positive and negative elevation trajectory (i.e. ± 30 deg), it is

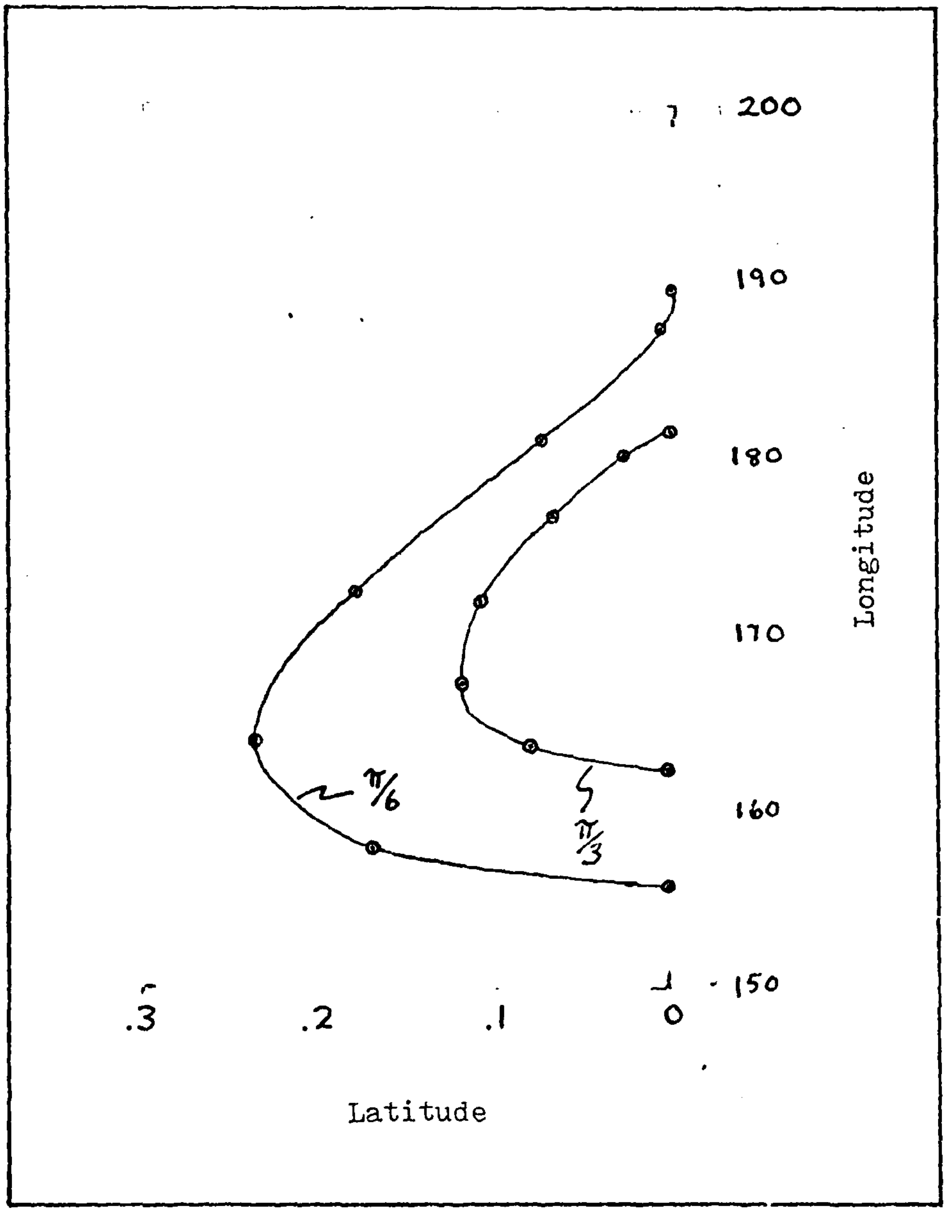

Fig. 8. Impact Envelopes

assumed that there are high and low trajectories to each impact point, excepting the case when the elevation is zero. This point is important in finding a probability for any impact point, since the probability density is composed of two probability densities summed.

In Fig 9, the maximum impact envelopes are plotted for five time decrements from orbit injection time. These represent each time step with the residual velocity vector elevation equal to zero. For the launch trajectory utilized in this study, failure results in a large dispersion in longitude and a relatively small dispersion in latitude. As the time decrements are subtracted, these dispersions grow. These characteristics would be expected in any launch configuration.

Probability Distribution

In an attempt to uncover curves of equal probability, the probability density was plotted versus the impact longitude. The result, for a specific time decrement and residual velocity vector elevation, is shown in Fig 10. A number of singularities were discovered in the probability density. These singularities represent areas of high probability.

The end singularities occur when the residual velocity vector azimuth is 0 and π radians. Close to these azimuths, a small change in residual velocity vector elevation results in a very small change in impact latitude, so there is a higher probability of impacting in those areas.

The middle singularity corresponds to an azimuth of $\frac{\pi}{2}$

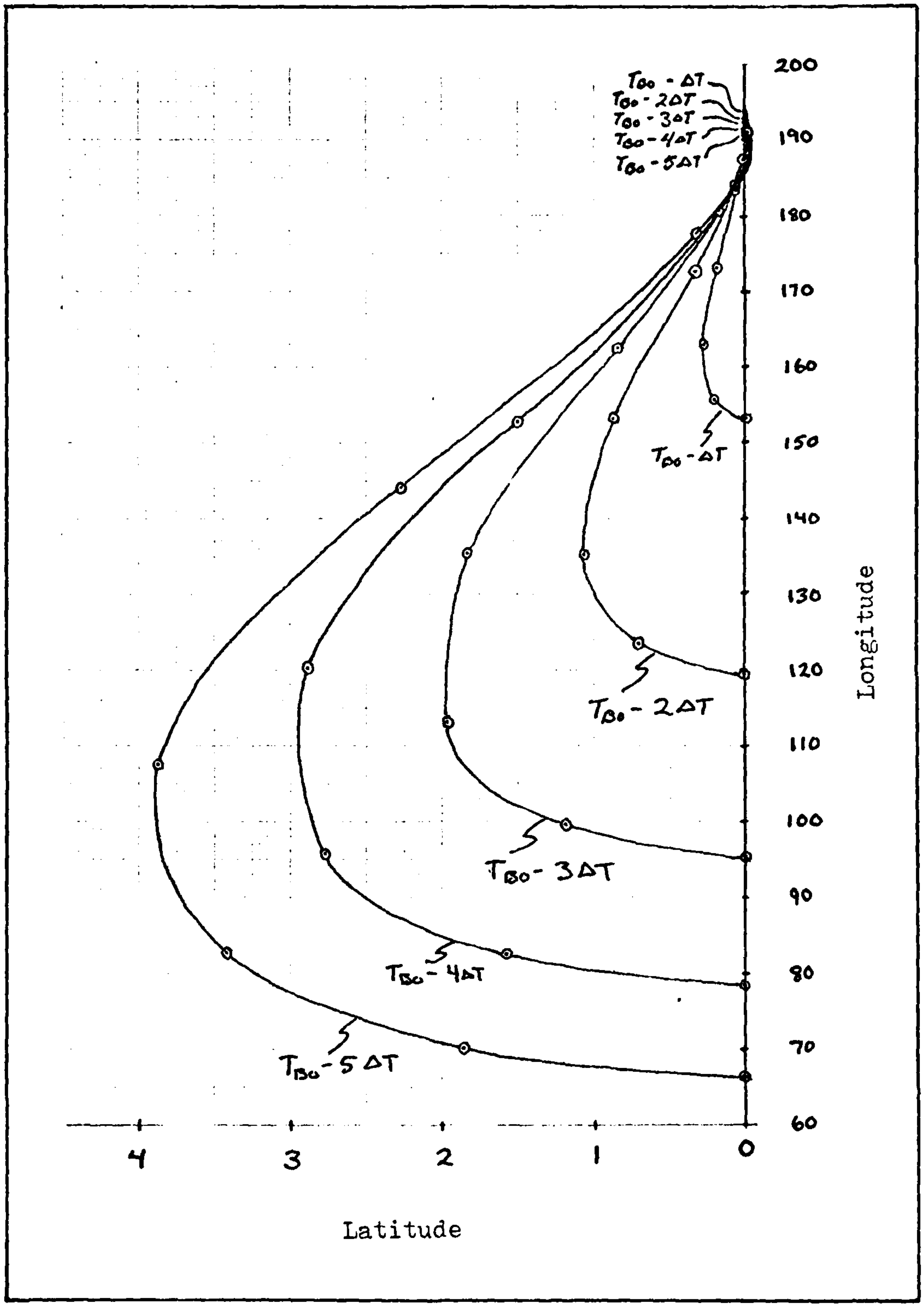

Fig. 9. Impact Envelopes with Changing Time of Failure

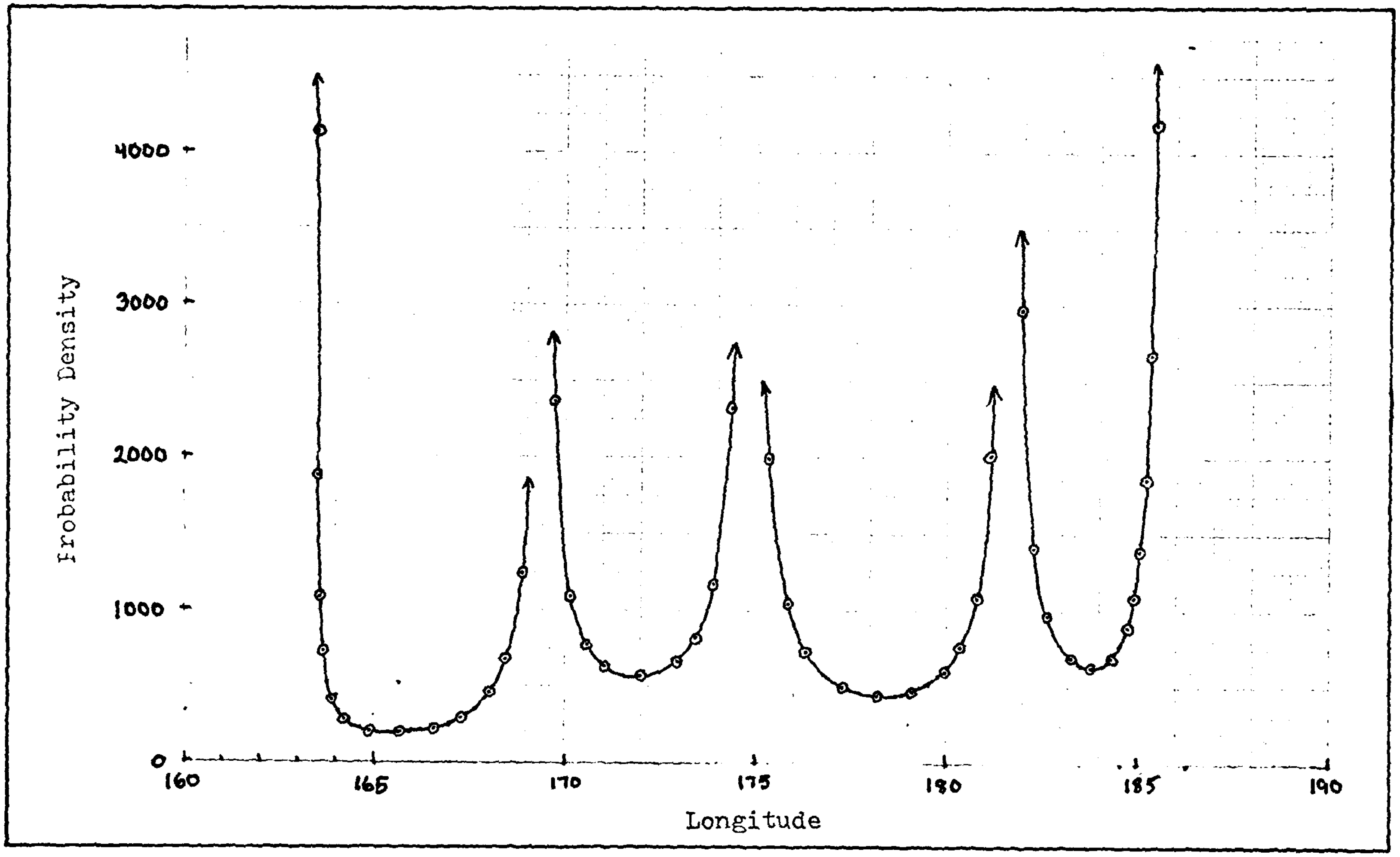

Fig. 10. Probability Density Along an Impact Envelope

radians. Two phenomena occur near this point. A small change in latitude, and a small change in azimuth result in a negligible change in longitude. These occurences give this area a high probability of impact.

The remaining singularities occur at maximum latitudes at set conditions. The singularity at ≈ 181.5 degrees corresponds to a maximum latitude for a constant azimuth of $\frac{5\pi}{18}$ radians. The singularity at ≈ 169.4 degrees corresponds to the maximum latitude for the elevation used for this plot. In these areas, a small change in elevation and azimuth, respectively, results in a negligible change in latitude, increasing the probability of impact. The position of the singularities along the impact curves, for the case in Fig 10, can be seen in Fig 11.

The number of singularities occuring can now be realized. Taking each burn time separately, for each set azimuth or elevation, there is a maximum latitude and longitude attainable, and a singularity corresponding to each. It would also be expected to have singularities corresponding to maximum latitudes and longitudes for a fixed residual velocity magnitude. All these singularities could be plotted, as in Fig 11, to obtain curves of high probability of impact for each time decrement. This is an area where further study would be enlightening, along with those mentioned in the next chapter on reccomendations.

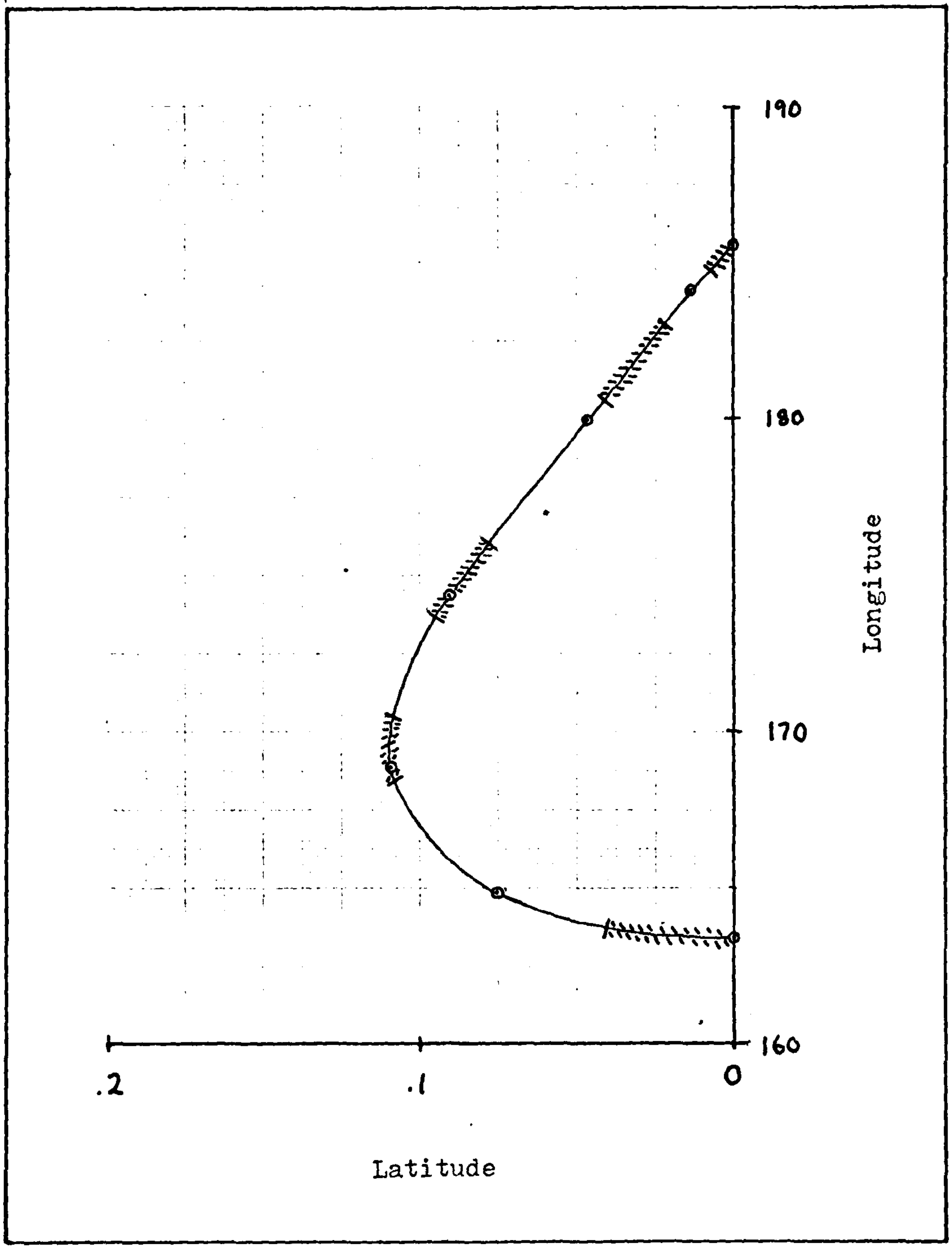

Fig. 11. Regions of High Probability Along an Impact Envelope

IV. Recommendations

There are many candidate areas for further study in this thesis. The scope of the study could be expanded to work with specific launch vehicles or launch sites. The launch equations of motion could be changed to incorporate a spherical earth, a changing gravitational acceleration, a non-constant thrust profile, a multi-stage vehicle, or flight through an atmosphere. The boundary value conditions could be changed to achieve an elliptical orbit.

As mentioned in the results chapter, the study could be taken deeper, without changing the scope, to find curves of high probability. Another approach to finding probability distributions would be to start at impact and work back to failure. This method would eliminate the problem of summing probabilities from near coincedent high and low trajectories. The depth in which the interested reader wishes to follow the study is certainly dependant on his desired results.

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Appendix A

Boundary Value Launch Computer Program

```
      PROGRAM LBV(INPUT=/80,OUTPUT)
      DIMENSION X(5),PSI(30),TI(30),DELM(30),DELV(30)
     +,TEMP(10),W(200),IW(5)
      EXTERNAL LEQ
      COMMON G,BETA,TH
      DELM(1)=0.0
      I=1
      M=0
   15 DM=0.0
      PRINT*," "
      PRINT*,"  DELM=",DELM(I)
   11 J=0
      L=0
      TB=1200.0
      G=32.17405
      RE=2.0923673E7
      GM=1.40734686E16
      PI=3.14159265536
   10 X(1)=0.0
      X(2)=0.0
      X(3)=.1
      X(4)=89.3*PI/180.0
      X(5)=21000.0/G+SQRT(DM**2)+DELM(I)
      PM=6.0*X(5)/7.0
      SM=X(5)-PM
      BETA=15.0/G
      TH=2.8*X(5)*G
      PRINT*," PARA",X(5),PM,SM,BETA,TH
      NEQN=5
      ABS=1.0E-8
      REL=1.0E-8
      EPS=1.0E-4
      IFLAG=1
      T=0.0
      IF(J.GT.0)GO TO 3
      TOUT=0.0
      DELT=TB/50.0
    1 TOUT=TOUT+DELT
      J=J+1
      CALL ODE(LEQ,NEQN,X,T,TOUT,REL,ABS,IFLAG,W,IW)
      IF(X(4).GT.0.0)GO TO 4
      IF(L.EQ.1)GO TO 4
      L=1
      DM=X(5)-SM
      IF(DM.LT.0.0)GO TO 11
      DM=0.0
    4 PSI(J)=X(4)
      TI(J)=TOUT
      IF(PSI(J).LT.-0.1) GO TO 2
```

```
      IF(TOUT.EQ.TB)GO TO 5
      GO TO 1
    5 IF(PSI(J).LT.0.0)GO TO 2
      GO TO 16
    2 K=4
      CALL INTERP(PSI,TI,J,K,X0,Y0,TEMP,IER)
      TF=Y0
      PRINT*," "
      PRINT*,"  TIME=",TF
      GO TO 10
    3 TOUT=TF
      CALL ODE(LEQ,NEQN,X,T,TOUT,REL,ABS,IFLAG,W,IW)
      PRINT*," X(I)",X(1),X(2),X(3),X(4),X(5),TOUT
      DVDM=-TH/(BETA-X(5))+G*SIN(X(4))/BETA
      V=SQRT(GM/(RE+X(2)))
      PRINT*,"V CIRCULAR=",V
      IF(M.EQ.1)GO TO 19
      PRINT*," I=",I
      DELV(I)=X(3)-V
      IF(I.EQ.25)GO TO 18
      IF(I.GT.1)GO TO 17
      DEM=-DELV(I)/DVDM
      I=I+1
      DELM(I)=DEM+DELM(I-1)
      GO TO 15
   17 I=I+1
      DELM(I)=DELM(I-1)+DEM
      GO TO 15
   18 CONTINUE
      CALL INTERP(DELV,DELM,I,K,X0,Y0,TEMP,IER)
      DELMF=Y0
      PRINT*," "
      PRINT*," DELTA MASS=",DELMF
      I=1
      M=1
      DELM(I)=DELMF
      GO TO 15
   16 PRINT*,"  PSI DOES NOT REACH 0"
   19 PRINT*," DONE"
      STOP
      END

      SUBROUTINE LEQ(T,X,XP)
      DIMENSION X(5),XP(5)
      COMMON G,BETA,TH
      XP(1)=X(3)*COS(X(4))
      XP(2)=X(3)*SIN(X(4))
      XP(3)=TH/X(5)-G*SIN(X(4))
      XP(4)=-G*COS(X(4))/X(3)
      XP(5)=-BETA
      RETURN
      END
```

Appendix B

Impact Point and Probability Prediction Computer Program

```
      PROGRAM FAIL(INPUT=/80,OUTPUT)
      DIMENSION X(5),XP(5),AZ(12),EL(12),DV(3),W(205),IW(5)
      COMMON G,BETA,TH,GM,PI,RE
      REAL LAT,LATDA,LATDE,LATI,LONG,LONGDA,LONGDE,LONGT,LONG2
      EXTERNAL LEQ
      K=0
      G=32.17405
      RE=2.0925673E7
      GM=1.40764688E15
      PI=3.1415926536
      READ*,TB,WE
      PRINT*," "
      PRINT*," TB=",TB," MASS=",WE
      BETA=15.0/G
      TH=2.8*WE*G
      PRINT*," BETA=",BETA," THRUST=",TH
      TOUT=TB
      IF(K.EQ.0)GO TO 20
   24 K=K+1
   20 IFLAG=1
      X(1)=0.0
      X(2)=0.0
      X(3)=.1
      X(4)=89.8*PI/180.0
      X(5)=WE
      NEQN=5
      AB=1.0E-8
      REL=AB
      T=0.0
      DELT=TB/100.0
      IF(K.EQ.0)GO TO 21
      TOUT=TOUT-DELT
   21 CALL ODE(LEQ,NEQN,X,T,TOUT,REL,AB,IFLAG,W,IW)
      PRINT*,"=================================================
     +========================================="
      PRINT*,"   RANGE          ALT           VEL          PSI
     +"          MASS            TIME"
      PRINT*,X(1),X(2),X(3),X(4),X(5),TOUT
      XP(1)=X(3)*COS(X(4))
      XP(2)=X(3)*SIN(X(4))
      PRINT*," XP1=",XP(1)," XP2=",XP(2)
      IF(K.GT.0)GO TO 25
      VCIRC=X(3)
      GO TO 24
   25 CONTINUE
      DELV=VCIRC-X(3)
      EL(1)=-PI/2.0
      AZ(1)=0.0
```

```
      DEL=PI/6.0
      DAZ=DEL
      DELAZ=1.0E-5
      DELEL=1.0E-5
      DO 23 I=1,6
      DO 22 J=1,12
      L=0
   26 CONTINUE
      DV(1)=DELV*COS(AZ(J))*COS(EL(I))
      DV(2)=DELV*SIN(AZ(J))*COS(EL(I))
      DV(3)=DELV*SIN(EL(I))
      IF(L.GT.0)GO TO 27
      PRINT*,"-------------------------------------------------
      PRINT*," AZ=",AZ(J),"     EL=",EL(I)
   27 CONTINUE
      CALL IMPACT(DV,X,XP,LAT,LONG,LONG2,DELT,_,ORBIT)
      IF(I.EQ.1)GO TO 32
      IF(AZ(J).EQ.0.0)GO TO 32
      EPS=.001
      ZING=(AZ(J)-PI)**2
      IF(ZING.LT.EPS)GO TO 32
      IF(ORBIT.GT.0)GO TO 31
      IF(L.EQ.1)GO TO 28
      IF(L.EQ.2)GO TO 29
      L=1
      LATI=LAT
      LONGI=LONG2
      AZI=AZ(J)
      ELI=EL(I)
      AZ(J)=AZI+DELAZ
      GO TO 26
   28 CONTINUE
      LATDA=LAT
      LONGDA=LONG2
      DADLA=DELAZ/(LATDA-LATI)
      DADLO=DELAZ/(LONGDA-LONGI)
      L=2
      AZ(J)=AZI
      EL(I)=ELI+DELEL
      GO TO 26
   29 CONTINUE
      LATDE=LAT
      LONGDE=LONG2
      DEDLA=DELEL/(LATDE-LATI)
      DEDLO=DELEL/(LONGDE-LONGI)
      EXP=ABS(COS(ELI))/(4.0*PI)
      P=ABS(DADLA*DEDLO-DADLO*DEDLA)*EXP
      EL(I)=ELI
   32 CONTINUE
      IF(L.GT.0)GO TO 33
      LATI=0.0
      P=0.0
   33 CONTINUE
      LATI=LATI*180.0/PI
```

```
      TIME=TOUT+DELT
      PRINT*," "
      PRINT*,"   LATITUDE      LONGITUDE    PROBABILITY     TIME"
      PRINT*,LATI,LONG,P,TIME
      IF(I.EQ.1)GO TO 23
   31 CONTINUE
   22 AZ(J+1)=AZ(J)+DAZ
   23 EL(I+1)=EL(I)+DEL
      IF(K.LT.5)GO TO 24
      PRINT*,"  DONE"
      STOP
      END

      SUBROUTINE LEQ(T,X,XP)
      DIMENSION X(5),XP(5)
      COMMON G,BETA,TH,GM,PI
      XP(1)=X(3)*COS(X(4))
      XP(2)=X(3)*SIN(X(4))
      XP(3)=TH/X(5)-G*SIN(X(4))
      XP(4)=-G*COS(X(4))/X(3)
      XP(5)=-BETA
      RETURN
      END

      SUBROUTINE IMPACT(DV,X,XP,LAT,LONG,LONG2,DELT,L,ORBIT)
      COMMON G,BETA,TH,GM,PI,RE
      DIMENSION V(3),R(3),H(3),EC(3),X(5),XP(5),DV(3)
      REAL MAGE,MAGR,MAGH,MAGV
      REAL LONG,LONG1,LONG2,LAT
      U=GM
      V(1)=DV(1)+XP(1)
      V(2)=DV(2)
      V(3)=DV(3)+XP(2)
      R(1)=X(1)
      R(2)=0.0
      R(3)=X(2)+RE
       H=R* V
      H(1)=R(1)*V(2)-R(2)*V(1)
      H(2)=R(2)*V(3)-R(3)*V(2)
      H(3)=R(3)*V(1)-R(1)*V(3)
      H2=0
      R2=0
      V2=0
      COEFV=0
      DO 11 I=1,3
      H2=H(I)**2+H2
      R2=R(I)**2+R2
      V2=V(I)**2+V2
   11 COEFV=R(I)*V(I)+COEFV
       GET ECCENTRICITY EC
      MAGH=SQRT(H2)
      MAGR=SQRT(R2)
      MAGV=SQRT(V2)
      COEFF=MAGV**2-U/MAGR
```

```
      E2=0
      DO 12 I=1,3
      EC(I)=(1.0/U)*(COEFR*R(I)-COEFV*V(I))
   12 E2=EC(I)**2+E2
      MAGE=SQRT(E2)
       TO FIND IF IT IMPACTS
      P=MAGH**2/U
      RPER=P/(1.0+MAGE)
      IF(RPER.GT.RE)GO TO 15
      A=P/(1.0-MAGE**2)
      COEFTNU=((P/MAGR)-1.0)/MAGE
      TANOM=ACOS(COEFTNU)
      IF(COEFV.GT.0.0)GO TO 16
      TANOM=2.0*PI-TANOM
   16 COEFENU=((P/RE)-1.0)/MAGE
      EANOM=ACOS(COEFENU)
      ANG=2.0*PI-TANOM-EANOM
      TANG=2.0*PI-EANOM
      E=ACOS((MAGE+COS(TANG))/(1.0+MAGE*COS(TANG)))
      E0=ACOS((MAGE+COS(TANOM))/(1.0+MAGE*COS(TANOM)))
      IF(TANG.LT.PI) GO TO 17
      E=2.0*PI-E
   17 IF(TANOM.LT.PI)GO TO 18
      E0=2.0*PI-E0
   18 CONTINUE
      DELT=SQRT(A**3/U)*((E-MAGE*SIN(E))-(E0-MAGE*SIN(E0)))
      AINC=ACOS(H(3)/MAGH)
      IF(V(2).GT.0.0)GO TO 8
      AINC=-AINC
    8 CONTINUE
      LONG1=ASIN(X(1)/RE)
      A180=PI
      IF(AINC.EQ.0.0)GO TO 9
      IF(AINC.EQ.A180)GO TO 9
      LAT=ASIN(SIN(AINC)*SIN(ANG))
      LONG2=ASIN(TAN(LAT)/TAN(AINC))
      CALL REDUCE(ANG,LONG2)
      GO TO 19
    9 CONTINUE
      LONG2=ANG
      LAT=0.0
   19 CONTINUE
      LONG=(LONG1+LONG2)*180.0/PI
      ANG=ANG*180.0/PI
      AINC=AINC*180.0/PI
      IF(L.GT.0)GO TO 10
      PRINT*," LAT=",LAT,"LONG=",LONG2
      PRINT*,"  "
      PRINT*,"    ANGLE            TIME            INCLINATION"
      PRINT*,ANG,DELT,AINC
   10 CONTINUE
      ORBIT=0.0
      GO TO 14
   15 PRINT*,"  ORBIT  MAG ECCENTRICITY=",MAGE
```

```
      ORBIT=1.0
   14 RETURN
      END
      SUBROUTINE REDUCE(ANG,LONG2)
      REAL LONG2
      PI=3.1415926535
      A90=PI/2.0
      A180=PI
      A270=1.5*PI
      A360=2.0*PI
      IF(ANG.LT.A360) GO TO 8
      ANI=ANG/A360
      IAN=INT(ANI)
      ANG=(ANI-IAN)*2.0*PI
    8 CONTINUE
      IF(ANG.LT.A90)GO TO 7
      IF(ANG.LT.A180)GO TO 6
      IF(ANG.LT.A270)GO TO 6
      LONG2=A360+LONG2
      GO TO 4
    7 LONG2=LONG2
      GO TO 4
    6 LONG2=A180-LONG2
    4 RETURN
      END
```

Vita

Scott W. Benson was born on 8 November 1955 in Geneva, Illinois. He graduated from Geneva Community High School in 1973. In May 1977, he received the degree of Bachelor of Science in Aeronautical and Astronautical Engineering from Purdue University. He received his commission through Air Force ROTC in May 1977. He entered the Air Force Institute of Technology in September 1977 as his first active duty assignment. Upon completion of his studies, he was assigned to the Foreign Technology Division, FTD/SDSY, at Wright-Patterson Air Force Base.

Permanent Address: 1022 James Street
Geneva, Illinois 60134

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The impact area and regions of high impact probability after booster failure were found for a sample space system. The impact footprints were found to be drop-shaped. They increased in size as the residual velocity vector elevation approached the original flight path, or when the booster failure was assumed earlier in the boost trajectory. Probability densities at impact were found, and plotted against the longitude of impact. Singularities in the probability density were discovered. The singularities correspond

to areas of high impact probability, and occur when a maximum latitude or longitude is obtained at a fixed residual velocity vector azimuth or elevation. These singularities were plotted along the impact envelopes. A sample failing booster system was used to demonstrate the impact area and probability characteristics.